Equilibrium Of Forces

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10:00 A.M - 1:00 P.M

1 Experimental Details:

1.1 Experiment 1

The apparatus of the first experiment consisted of three masses, two of which were attached to Pulleys using twine that was connected to a central ring which also had the third mass attached with twine. (See Fig, 1.1)

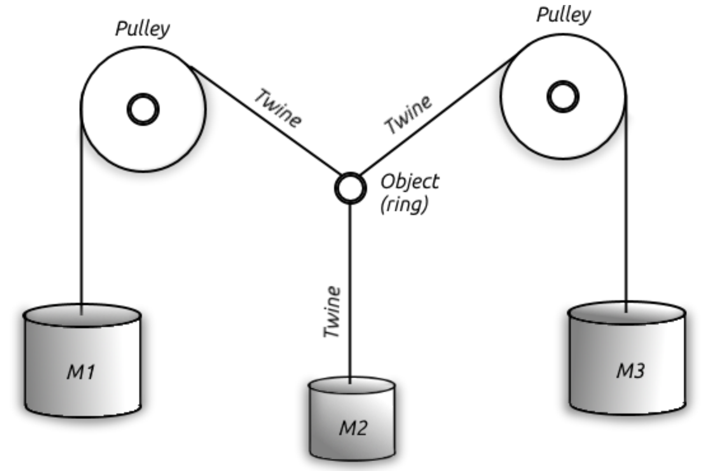

Figure 1.1: Apparatus Diagram .

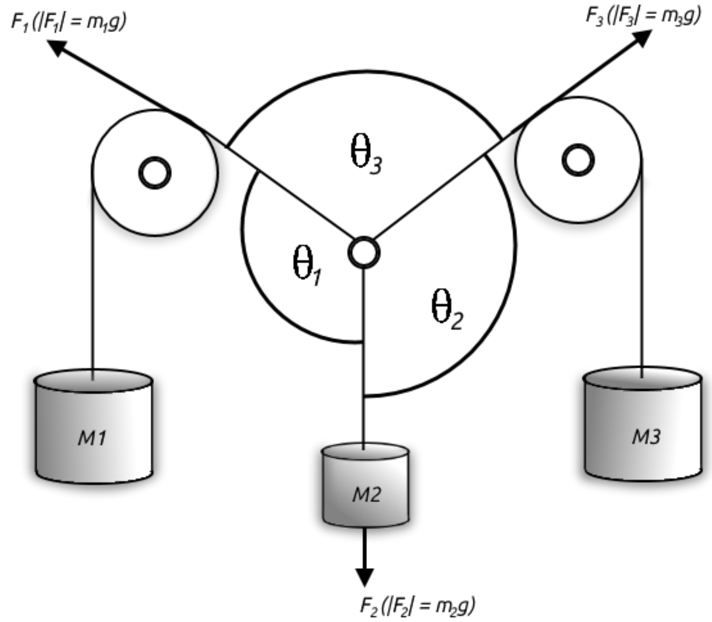

Figure 1.2: Apparatus with vectors ($\vec{v}$) & angles (θ)

The important measurements of this experiment are the three masses: (M1,M2 and M3) which are used to help work out the three vector forces: (F_1, F_2 and F_3) and the three angles: (θ_1,θ_2 and θ_3). (See Fig, 1.2).

1.2 Experiment 2

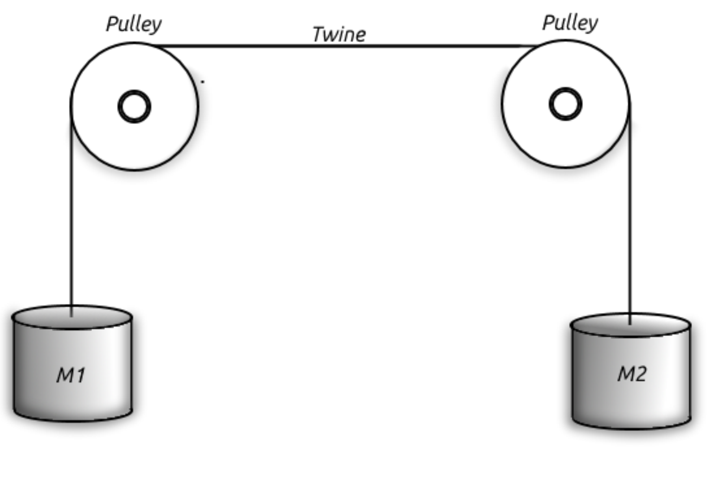

Figure 1.3: Apparatus Diagram .

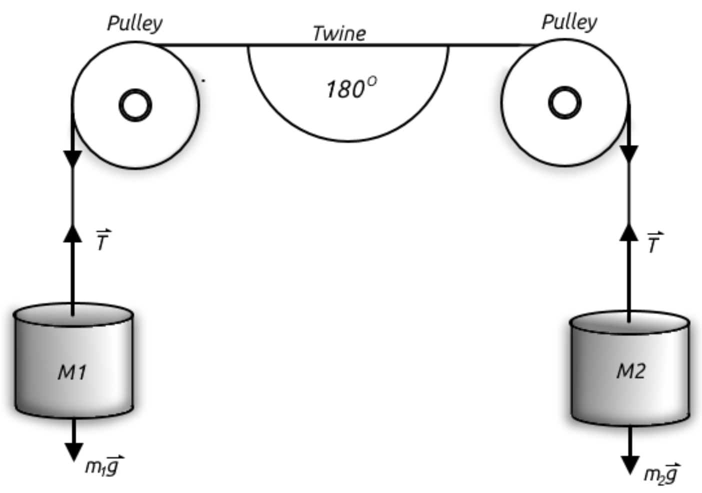

Figure 1.4: Apparatus with vectors ($\vec{v}$) & angles (θ)

The apparatus for the second experiment consisted of two masses, both attached to Pulleys using string so that the system was in equilibrium. (See Fig 1.3)

The important measurements of this experiment are the two masses (M1 and M2) which are used to work out the vector forces ($m\vec{g}$) and it is important to ensure the string is perfectly horizontal which means it has an angle of 180°.

2 Results and Discussion:

Table 1: Results of experiment 1:

Mass 1 (kg)	Mass 2 (kg)	Mass 3 (kg)	$\theta_1(°)$	$\theta_2(°)$	$\theta_3(°)$
0.075	0.055	0.105	110	110	140
0.13	0.155	0.125	125	130	105

$$|F_1| = m_1 g, \quad 0.73575 = (0.075)(9.81)$$
$$F_x = F cos\theta, \quad -^{*}0.69 = (0.73575)cos(20°)$$
$$F_y = F sin\theta, \quad 0.25 = (0.73575)sin(20°)$$

$$|F_2| = m_2 g, \quad 0.53955 = (0.055)(9.81)$$
$$F_x = 0, \;\; 0 = (0.53955)cos(90°)$$
$$F_y = -0.54, \;\; -^{*}0.54 = (0.53955)sin(90°)$$

$$|F_3| = m_3 g, \; 1.03005 = (0.105)(9.81)$$
$$F_x = 0.97, \;\; 0.97 = (1.03005)cos(20°)$$
$$F_y = 0.35, \;\; 0.35 = (1.03005)sin(20°)$$

$$|F_1| = m_1 g, \quad 1.2753 = (0.13)(9.81)$$
$$F_x = F cos\theta, \quad -^{*}1.04 = (1.2753)cos(35°)$$
$$F_y = F sin\theta, \quad 0.73 = (1.2753)sin(35°)$$

$$|F_2| = m_2 g, \quad 1.52055 = (0.155)(9.81)$$
$$F_x = 0, \;\; 0 = (1.52055)cos(90°)$$
$$F_y = -1.52, \;\; -^{*}1.52 = (1.52055)sin(90°)$$

$$|F_3| = m_3 g, \; 1.22625 = (0.125)(9.81)$$
$$F_x = 0.94, \;\; 0.94 = (1.22625)cos(40°)$$
$$F_y = 0.79, \;\; 0.79 = (1.22625)sin(40°)$$

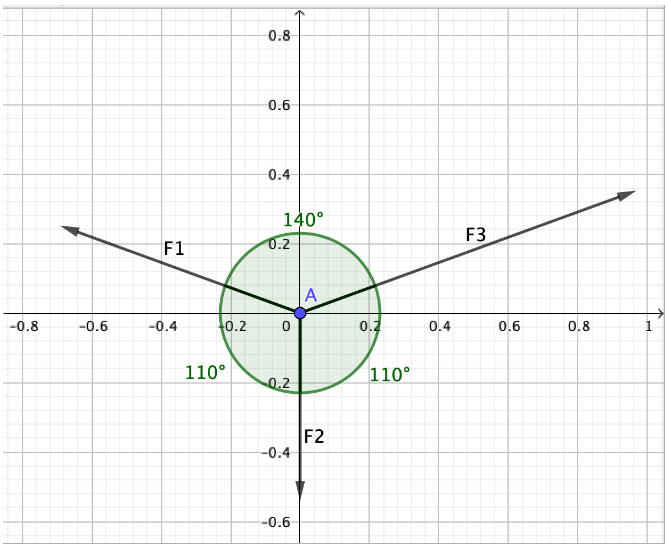

Figure 2.1: Vector Graph 1 .

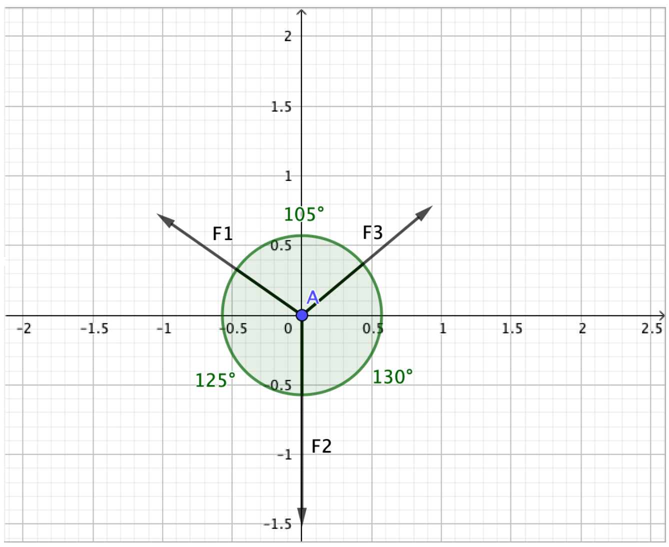

Figure 2.2: Vector Graph 2

It should be noted that $(-^{*})$ is used to represent values that are positive in our calculations but are made negative due to the vectors direction on the Cartesian plane.

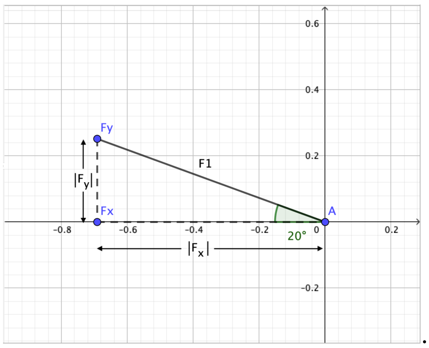

Figure 2.3: Co-Ordinate F1

To calculate the angle, we needed to use our vector diagram to determine what angle F1 made with the x-axis, using θ_1 which was 110° we took away the right angle which was in the negative y direction and got $110° - 90° = 20°$. This new angle is the one we used for our calculations since we know F1 = 0.73575 and $\theta = 20°$

$$F_{\mathrm{x}} = (0.73575)cos(20°)$$
$$F_{\mathrm{y}} = (0.73575)sin(20°)$$

Which gave us an F_{x} value of 0.69 and an F_{y} value of 0.25, We know that our F1 vector was left and therefore we changed our F_{x} value to a negative.

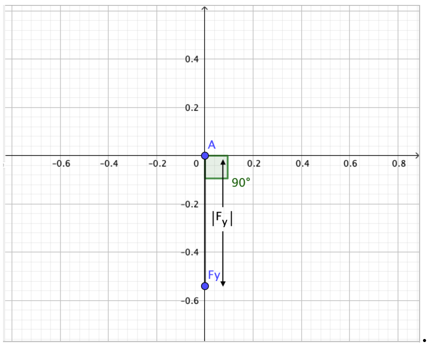

Figure 2.4: Co-Ordinate F2

It was much easier to calculate this angle, as the mass was hanging perfectly vertically downwards, our x value is always going to be zero and the angle our vector makes with the x axis is 90° as it is perfectly perpendicular. We know F2 = 0.53955 and $\theta = 90°$

$$F_{\mathrm{x}} = (0.53955)cos(90°)$$
$$F_{\mathrm{y}} = (0.53955)sin(90°)$$

Which gave us an F_{x} value of 0 and an F_{y} value of 0.54, We know that our F2 vector was downward and therefore we changed our F_{y} value to a negative.

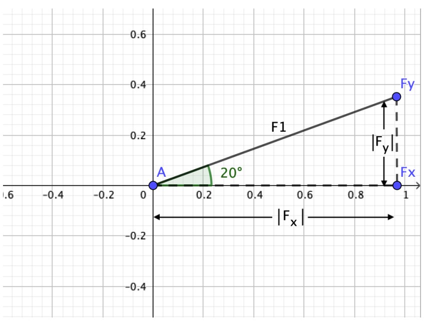

Figure 2.5: Co-Ordinate F3

We then repeat this one more time for F3, since we know that θ_1 and θ_2 were the same angles we already know that our angle is going to be 20°, and we know F3 = 1.03005.

$$F_{\mathrm{x}} = (1.03005)cos(20°)$$
$$F_{\mathrm{y}} = (1.03005)sin(20°)$$

Which gave us an F_{x} value of 0.97 and an F_{y} value of 0.35, both of which already have the correct direction.

We then repeat this process again, this time calculating the Co-Ordinates of our second vector graph.

We used our vector diagram to determine what angle F1 made with the x-axis, using θ_1 which was 125° we took away the right angle which was in the negative y direction and got $125° - 90° = 35°$. This new angle is the one we used for our calculations since we know $F1 = 1.2753$ and $\theta = 35°$

$$F_{\mathrm{x}} = (1.2753)cos(35°)$$
$$F_{\mathrm{y}} = (1.2753)sin(35°)$$

Which gave us an F_{x} value of 1.04 and an F_{y} value of 0.73, We know that our F1 vector was left and therefore we changed our F_{x} value to a negative.

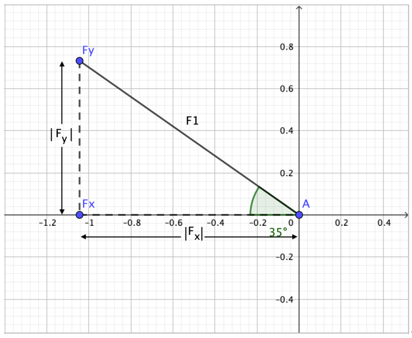

Figure 2.6: Co-Ordinate F1

Once again, It was much easier to calculate this angle, as the mass was hanging perfectly vertically downwards, our x value is always going to be zero and the angle our vector makes with the x axis is 90° as it is perfectly perpendicular. We know $F2 = 1.52055$ and $\theta = 90°$

$$F_{\mathrm{x}} = (1.52055)cos(90°)$$
$$F_{\mathrm{y}} = (1.52055)sin(90°)$$

Which gave us an F_{x} value of 0 and an F_{y} value of 1.52, We know that our F2 vector was downward and therefore we changed our F_{y} value to a negative.

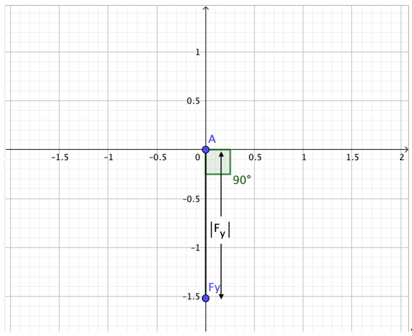

Figure 2.7: Co-Ordinate F2

This time we know that θ_1 and θ_2 are not the same angles, so we have to use θ_2 and once again subtract the right angle below the x-axis, therefore we do $130° - 90° = 40°$. Now we know that our angle is 40°, and $F3 = 1.22625$.

$$F_{\mathrm{x}} = (1.22625)cos(40°)$$
$$F_{\mathrm{y}} = (1.22625)sin(40°)$$

Which gave us an F_{x} value of 0.94 and an F_{y} value of 0.79, both of which already have the correct direction.

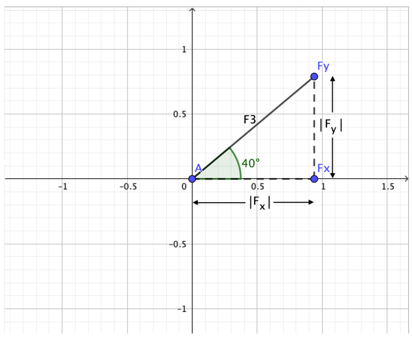

Figure 2.8: Co-Ordinate F3

2.1 Polygon method & Components method:

The concept the Polygon method is to place the vectors end to end in order to visually see how they affect each other. Using the Polygon method on our first vector diagram we get this:

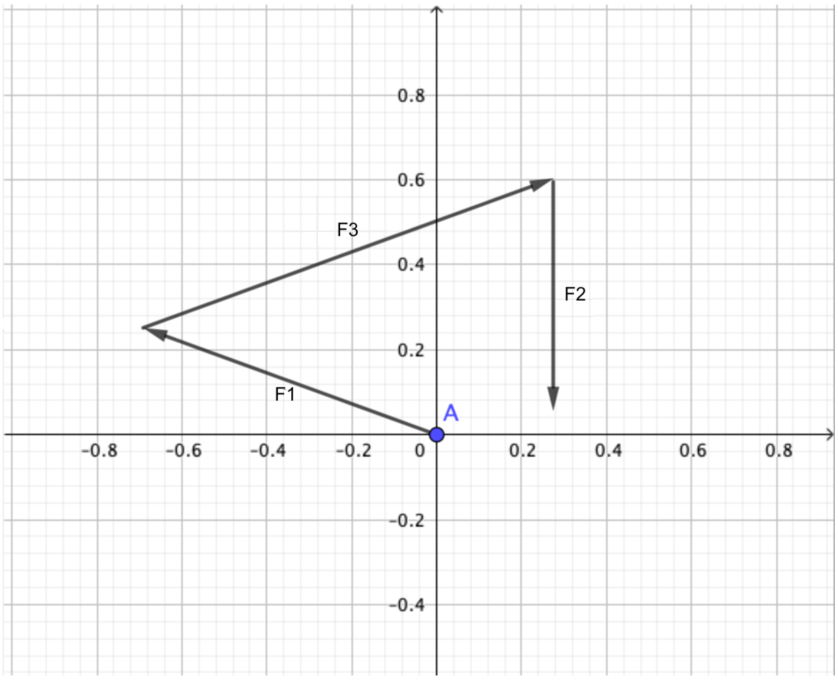

Figure 2.9: Polygon Diagram of Vector 1

Using the polygon vector on our second vector diagram we get:

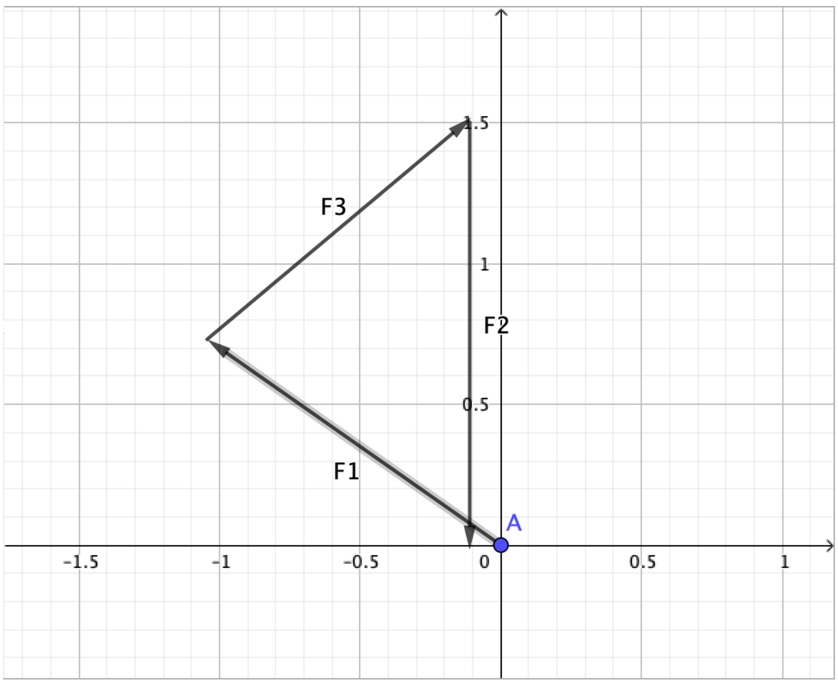

Figure 2.10: Polygon Diagram of Vector 2

As we can see our vectors almost perfectly align to (0,0), however it is slightly off.

We can use the component method to workout how off our vectors are from (0,0) mathematically: Using the component equation involves adding all our x values together like so:

$$1F_{\mathrm{x}} + 2F_{\mathrm{x}} + 3F_{\mathrm{x}} + ... \tag{1}$$

Since we only have two F_x values (since F2's F_x value is 0) we only have to add the two of them.

$$\text{Vector 1: } -0.69 + 0.97 = 0.28$$

$$\text{Vector 2: } -1.04 + 0.94 = -0.1$$

Next we have to add all of the y values together using:

$$1F_{\mathrm{y}} + 2F_{\mathrm{y}} + 3F_{\mathrm{y}} + ... \tag{2}$$

Since we have three F_y values, we must add all three of them. Vector 1 and Vector 3 are both positive in the y-axis and Vector 2 is negative.

$$\text{Vector 1: } 0.25 + 0.35 - 0.54 = 0.06$$

$$\text{Vector 2: } 0.73 + 0.79 - 1.52 = 0$$

Now that we've finished these calculations we can determine how far from (0,0) our values are by getting the x and y co-ordinates for each vector:

$$\text{Vector 1: } (0.28, 0.006)$$

$$\text{Vector 2: } (-0.1, 0)$$

As we can see, the vector forces that would cause these co-ordinates would be extremely small and it is highly likely that these discrepancies are the result of unaccountable external forces acting on the system and reducing our accuracy. If we could carry out this experiment in a vacuum without the influence of external forces these co-ordinates would most likely reduce to (0,0) as expected.

2.2 Experiment 2

Table 2: Results of experiment 2 (Adding 2 grams to equilibrium system):

	Unblocked:	Blocked:
Run 1	0.00	2.44
Run 2	0.00	2.04
Run 3	0.00	1.98
Run 4	0.00	2.03
Run 5	0.00	1.95
Run 6	0.00	1.93
Run 7	0.00	2.11
Run 8	0.00	2.10
Run 9	0.00	1.89
Run 10	0.00	2.58

Table 3: Results of experiment 2 (Adding 20 grams to equilibrium system):

	Unblocked:	Blocked:
Run 1	0.00	0.46
Run 2	0.00	0.44
Run 3	0.00	0.49
Run 4	0.00	0.50
Run 5	0.00	0.45
Run 6	0.00	0.46
Run 7	0.00	0.50
Run 8	0.00	0.51
Run 9	0.00	0.42
Run 10	0.00	0.47

Table 4: Measurements of experiment 2 used:

Distance between lightgates	Added Mass (g)
15 cm	2
15 cm	20

(i) How much extra mass is required to cause acceleration?
2 grams of extra force resulted in the system leaving equilibrium and caused the mass to accelerate.
why isn't motion observed as soon as $m_2 > m_1$?
Ideally if the pulley system existed without the affect of any external forces acting on it, we would see the system leave equilibrium as soon as there was any change in either of the two masses, however due to external forces such as friction playing into the experiment, this did not occur until at least 2 grams of force was applied to one side of the system.

Can we determine contribution of frictional forces?
Since we know that any amount of external force 'should' ideally make the system lose equilibrium and that in our experiment we require 2 grams of weight in order for this to occur, we can determine the amount of frictional forces at play using the equation:

$$F = m\ a \tag{3}$$

Where our mass (m) is the 2 grams (0.002 kg) required to cause the system to leave equilibrium, and acceleration (a) is the acceleration due to gravity (9.81 ms^{-2}).
Using Eq.1 gives us:

$$0.01962 = (0.002)(9.81)$$

We have then determined that there is roughly 0.01962 Newtons (N) of frictional force acting on the system which it has to overcome before it will leave equilibrium.

(ii) Calculate tension in the string T for the equilibrium and out of equilibrium situations
We can determine the tension in the strings using:

$$T = 2\frac{m_1 m_2}{m_1+m_2}g \tag{4}$$

Using Eq. 2 for our string in equilibrium we get:

$$0.981 = (2\frac{(0.1)(0.1)}{(0.1)+(0.1)})(9.81)$$

Therefore we have 0.981N of force acting as tension on the string.

Using Eq. 2 for our string out of equilibrium we get:

$$0.991 = (2\frac{(0.1)(0.102)}{(0.1)+(0.102)})(9.81)$$

Therefore we have 0.991N of force acting as tension on the string.

(iii) Calculate acceleration a for the two masses $m_1 m_2$ using the equation of motion $y = \frac{1}{2}at^2$. Calculate experimental tension using $T = m_2(g - a)$
Using the equation:

$$y = \frac{1}{2}at^2 \tag{5}$$

We can work out a value for a since we know y = distance between timegates and we use our average times for t. Manipulating Eq. 3 we get:

$$\frac{(2)0.15}{2.105^2} = 0.07ms$$

and

$$\frac{(2)0.15}{0.47^2} = 1.36ms$$

We can calculate experimental Tension using:

$$T = m_2(g - a) \tag{6}$$

Using Eq. 4 we get:

$$0.974 = (0.1)(9.81 - 0.07)$$

and

$$0.993 = (0.102)(9.81 - 0.07)$$

Therefore we have 0.974N and 0.993N of force acting as tension on the string respectively.

(iv) Compare experimental values of T and a with theoretical values of T and a. Discuss results and give conclusion
To determine the difference between our two values for T we can use the equation:

$$\frac{\mid Theoretical\,Value - Experimental\,Value \mid}{TheoreticalValue} x100 \tag{7}$$

$$\frac{\mid 0.981 - 0.974 \mid}{0.981} x100 = 0.71\%$$

$$\frac{\mid 0.991 - 0.993 \mid}{0.991} x100 = 0.20\%$$

The difference between our theoretical values and experimental values are $< 1\%$

Our theoretical acceleration can be calculated using the equation:

$$a = \frac{m_2 - m_1}{m_2 + m_1} g \tag{8}$$

Using Eq. 6 we get:

$$0.10ms = \frac{0.102-0.1}{0.102+0.1}(9.81)$$

and

$$0.89ms = \frac{0.12-0.1}{0.12+0.1}(9.81)$$

We can once again use Eq. 5 to determine the difference:

$$\frac{\mid 0.10 - 0.07 \mid}{0.10} x100 = 30\%$$

and

$$\frac{\mid 0.89 - 1.36 \mid}{0.89} x100 = 52.81\%$$

Our Theoretical and Experimental values for Acceleration differ much greater than our values for Tension did.

Interestingly, in both circumstances we got one value where the theoretical was higher and one value where the experimental was higher.

3 Conclusion

We have determined that a state of equilibrium in a system is the product of all vector forces acting on that system adding to zero, furthermore a system in equilibrium will leave equilibrium if an external forces is presented unevenly to the vectors, however despite theoretically any amount of force resulting in the loss of equilibrium- in the lab there is a threshold amount of force required to cause equilibrium to be lost. This is believed to be caused by the external forces which aren't accounted for within the system, most notably friction.

If this experiment was to be reproduced it would be beneficial to carry out the experiment getting more values for a wider range of masses and angles as these extra values will help increase the overall accuracy of the results and minimise any errors.